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Robert Malcolm Fortson, Jr.

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Instrument Approach Control and Runway Requirements

for

an Advance Base Area

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INSTRUMENT APPROACH CONTROL AND RUNWAY REQUIREMENTS FOR AN ADVANCE BASE AREA

by

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PREFACE

This Thesis is concerned with the determination of the number of instrument approach controls and runways required to support a military advance base during a time of armed conflict. The solution outlined is regarded as being of interest to Naval Officers responsible for logistics planning. It points out how operational requirements can lead to the determination of a lower limit on the facilities needed to satisfy the air operations phase of the operational requirements of aircraft units. The berthing facilities are not considered here since current logistics planning material covers these aspects in considerable detail.

The writer's interest in the problem stems from a tour of duty as the Air Traffic Control Officer at an overseas airbase. During this tour the capabilities of a runway and instrument approach control proved to be a very controversial topic. A Naval logistic planner, poorly estimating these capabilities, could be the cause of a completely unbalanced logistic plan for a given air operation requirement. The minimization of this possibility is the prime aim of this paper.

This Thesis was written at the U.S. Naval Postgraduate School, Monterey, California, during January - May 1957. The writer is indebted to Professor Thomas E. Oberbeck for his assistance in the early phases; to Professor Charles C. Torrance for his most helpful critisms and guidance as

first reader; and to Professor C. A. Magwire for his valuable assistance as second reader. Appreciation is expressed to Commander C. P. Yonkers, USN and T. A. Wright, ATC, USN for data they provided on aircraft landing and take-off times.

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TABLE OF SYMBOLS

E_x	=	one of the set of possible discrete states of a system, where x denotes the number of aircraft being serviced or waiting for service.
$\lambda(t)dt$	=	Probability that a system in state E_x will go to state E_{x+1} during the time interval t to t+dt. It is implied that $\lambda(t)$ = mean arrival rate. ($\lambda(t)$ is independent of x.)
$\mu(t)dt$	=	probability that a system in state E_x will go to state E_{x-1} during the time interval t to t-dt. It is implied that $\mu(t)$ = mean service rate. ($\mu(t)$ is independent of x.)
$\rho(t)$	=	traffic density of the system (i.e. $\rho(t) = \lambda(t)/n \cdot \mu(t)$ where n is the number of service facilities).
$P_{i,x}(t)$	=	probability that the system is in state x at time t when the system started in state i.
$M_i(t)$	=	the expected state of the system at time t when the system started in state i.
$F(\rho)$	=	probability that an arrival will have to wait at least a time τ before being serviced when the state of the system is not E_o.
P/τ	=	probability that an arrival will have to wait for a period longer than τ.
τ_w	=	expected waiting time.
$I_k(z)$	=	modified Bessel Function of the first kind.
τ_e	=	time that the facility is closed by an emergency.
τ_m	=	the maximum expected waiting time caused by an emergency of length τ_e.
$M_w(t_e)$	=	expected number of aircraft waiting for a landing when the runway is cleared at time t_e and when w were waiting when the facility was closed at time t.
w	=	expected number waiting under normal operating conditions.

$f(t)$ = the density function for a designated service time distribution for a flight of aircraft (e.g., $f_{jT}(t)$ = the density function for jet aircraft take-off times) with $E(t)$ and σ_t as the mean and standard deviation of the distribution.

CHAPTER I

INTRODUCTION

This thesis is concerned with a study of the air operations of an advanced airbase in a period of national emergency. It shows how the military logistics planner can utilize the results obtained in the study to assist him in formulating his overall logistic plan. The thesis is directed toward the military planner, on the Area Commander's staff, who is formulating his logistics requirements for future planned military advances within a rather limited time period.

By air operations is meant (a) the take-off and landing of aircraft when there is good weather around the airbase, or (b) the departure and approach of aircraft when there is instrument weather around the airbase. Good weather, as used here, means that the visibility is such that the pilot is able to guide his aircraft to and from the airbase safely; and instrument weather means that the visibility is so reduced that the pilot relies on other aids to guide his aircraft to and from the airbase. The term departure, as used in this paper, means the aircraft take-off and flight of the aircraft out of the airbase area when the airbase has instrument weather. The term approach, as used here, means the flight of the aircraft to the airbase and its landing when the airbase has instrument weather.

Prior to the development of an advanced base plan, the logistic planner must consider such items as:

1. The mission assigned to the advanced base development, which must include the readiness schedule for facilities.

2. The site requirements, which are closely related to the mission, and many special items such as substance of the ground, weather condition, location of nearby water, communications, and nearby water front unloading area.

3. The degree of permanency of the facilities.

4. The area available for development.

5. Local resources available to reduce requirements.

6. The priority of the development of the facilities.

For a more detailed discussion of the basic considerations involved in planning and developing advanced bases, the reader is directed to N W P 11-23. [1]

This thesis takes the information given to the logistic planner (the missions of various units) and from it determines the minimum number of airbase instrument approach controls and the minimum number of airbase runways that should be provided at the advanced base area. An airbase instrument approach control consists of the airspace, electronic aides and other items needed to guide aircraft to and from the airbase runway in instrument weather conditions. With this result the planner can continue with the problems of site location and other items, knowing what sites are

needed to just accomplish the mission. The procedure suggested here greatly reduces the possibility of having an airbase that cannot meet the operational flight requirements of the aircraft that it is able to support logistically.

CHAPTER II

DEVELOPMENT OF THE BASIC MODEL

An airbase can be described by the number of aircraft actually being serviced or the number waiting for service. The term service includes take-off, landing, approach and departure; and the service facility consists of the runways, or the runways and the approach control, depending on the weather condition at the airbase. With this in mind, a model of the air operations of an airbase may be formulated using the following notations:

E_x = one of the set of possible discrete states of a system, where x denotes the number of aircraft being serviced or waiting for service.

$\lambda(t)dt$ = probability that a system in state E_x will go to state E_{x+1} during the time interval t to t+dt. It is implied that $\lambda(t)$ = mean arrival rate. ($\lambda(t)$ is independent of x.)

$\mu(t)dt$ = probability that a system in state E_x will go to state E_{x-1} during the time interval t to t+dt. It is implied that $\mu(t)$ = mean service rate. ($\mu(t)$ is independent of x.)

$\rho(t)$ = traffic density of the system (i.e., $\rho(t) = \lambda(t)/n \cdot \mu(t)$ where n is the number of service facilities).

$P_{i,x}(t)$ = probability that the system is in state x at time t when the system started in state i.

$M_i(t)$ = the expected state of the system at time t when the system started in state i.

$F(\rho)$ = probability that an arrival will have to wait at least a time τ before being serviced when the stateof the system is not E_o.

P/τ = probability that an arrival will have to wait for a period longer than τ.

τ_w = expected waiting time.

The following set of iteration formulae can be derived [1]:

$$P'_{i,o}(t) = -\lambda(t) \cdot P_{i,o}(t) + \mu(t) \cdot P_{i,1}(t) \qquad (1)$$

$$P'_{i,x}(t) = -\left\{\lambda(t) + x\mu(t)\right\} P_{i,x}(t) + \lambda(t) \cdot P_{i,x-1}(t) + (x+1)\mu(t) \cdot P_{i,x+1}(t)$$
$$\text{for } 0 \leq x < n, \qquad (2)$$

$$P'_{i,x}(t) = -\left\{\lambda(t) + n\mu(t)\right\} P_{i,x}(t) + \lambda(t) P_{i,x-1}(t) + n\mu(t) \cdot P_{i,x+1}(t)$$
$$\text{for } n \leq x. \qquad (3)$$

Here $P'_{i,j}(t)$ is the time derivative of $P_{i,j}(t)$.

Assume that:

1. There is a Poisson distribution of arrivals.

2. There is an exponential distribution of service time (i.e., the probability that the service time will terminate

during the next time period is independent of the length of the present service period).

3. The traffic density is less than one.

The solution to Equation (2) and (3) under these conditions was first obtained by Erlang, an employee of a Danish telephone company, in 1917. The solution in terms of the expected waiting time when $n = 1$ is:

$$\tau_w = \frac{\lambda}{\mu(\mu - \lambda)} \tag{4}$$

The same result expressed in terms of the probability that units arriving will have to wait at least a time τ before being served is

$$P/\tau = P/o \cdot F(\rho), \tag{5}$$

here

$$P/o = \frac{(n\rho)^n}{n!(1-\rho)} \left\{ \frac{1}{1 - n\rho + \frac{(n\rho)^2}{2!} + \cdots \frac{(n\rho)^{n-1}}{(n-1)!} + \frac{(n\rho)^n}{n!(1-\rho)}} \right\}$$

and

$$F(\rho) = \exp\left\{-n(1-\rho)\tau\right\}.$$

A detailed derivation of Equation (4) has been given by Churchman, Achoff, and Arnoff [2]. A summary of the derivation of Equation (5) is given by Molina [3]. Graphs of P/o for $n=1$ and 2, and a graph $F(\rho)$ for several values of ρ have been extracted from a collection of curves and graphs published by the Port of New York Authority [4] and are contained as Appendix I.

A solution to the general Equation (1), (2), and (3) has been obtained by A. B. Clarke [5] for the time dependent case and a multi-channel service mechanism using the assumption of Poisson arrivals, exponential services times and ρ, traffic density, restricted only to the extent that it be a positive constant. This solution is:

$$P_{i,x}(t) = \frac{1}{x!} \exp\left\{ -\frac{\lambda}{\mu}(1 - \exp -\mu \cdot t) \sum_{k=0}^{x} \binom{x}{k} \frac{i!}{(i-k)!} \right.$$

$$\left. \frac{\lambda}{\mu}^{x-k} \left(1 - \exp(-\mu t)^{x+i-2k} \exp(-\mu tk) \right) \right\} \quad (6)$$

$$\text{for } 0 \leq x < n,$$

$$P_{i,x}(t) = \exp - (\lambda + n\mu)t \left\{ (\rho)^{\frac{x-i}{2}} I_{i-x}(2t\sqrt{\lambda n \mu}) \right.$$

$$+ (\rho)^{\frac{x-i-1}{2}} I_{i+1+x}(2t\sqrt{\lambda n \mu})$$

$$+ (1-\rho)(\rho)^{x} \sum_{k=i+x+2}^{\infty} (\rho)^{-\frac{k}{2}} I_{k}(2t\sqrt{\lambda n \mu}) \quad (7)$$

$$\text{for } x \geq n,$$

where $I_k(z)$ is the modified Bessel function of the first kind.

Morse [6] has solved the system with the same arrival and service assumptions as Clarke except that $0 \leq \rho < 1$. The solution consists of corrections to be applied to the

stationary state probabilities, $P_{i,x}(\infty)$. The expression for the correction term requires computations similar to those required by Equation (7) above. Luchak [7] solved the equation for the assumptions $\rho =$ positive constant less than one, arrivals are random, and service time is a Pearson Type III Distribution. The Pearson Type III Distribution reduces to a negative exponential distribution as a special case. This solution requires computations similar to Equation (7) above, but involves a new Bessel type function.

The range of values that can be examined by any of the three solutions (Clarke's, Morse's or Luchak's) are restricted by the lack of tables on the Bessel type functions encountered. From the numerical results obtained by Clarke [5] for $P_{o,x}(t)$, a plot of $M_o(t)$ was constructed and is attached in Appendix II. The solution for the transient Equations (6) and (7), and for the steady state Equations (4) and (5), will be used in the problem under consideration. These two models do not, however, represent the total system. Greater use might have been made of the transient solution had tables of the Bessel type functions, $I_k(z)$, for large values of "k" been available. The cases where these two models are not used will be handled using basic probability concepts. Each such case will be covered when it is reached as the basic model is applied to the actual system in Chapter IV.

CHAPTER III

CHARACTERISTICS OF AIRBASE PARAMETERS

Before applying the model described in Chapter II to advanced airbase air operations system, the following characteristics of the actual system will be discussed: service time distribution, $1/\mu(t)$; distribution of the time interval between successive arrivals (arrival interval distribution), $1/\lambda(t)$; and the traffic handling priorities.

Service Time Distribution

Data were collected of aircraft take-offs and landing times at the U.S. Naval Air Station Moffett, California, to obtain information on the service time distribution of present day operational naval aircraft. A brief summary of the data collected, with histograms, is contained in Appendix III. It was found that the landing and take-off times of the operational jet carrier aircraft (FJ, F9F-6, A4D, and F3H) were distributed in about the same manner. The timings of these aircraft were grouped together, and a distribution for jet aircraft landing and jet aircraft take-off were obtained (see Appendix III, page 41).

It was noticed that the take-off time of jet aircraft in flights of two required about the same total overall time as a jet aircraft departing alone; also for landings in flights of two the total time for the pair of aircraft

was about one third longer than the landing time of an individual aircraft.

In the two flights observed in which flights of four aircraft departed as a group, the total time was about twice that for a flight of two. This is understandable when it is realized that the third and fourth aircraft of the flight departed as a group of two, closely following the first group of two. (The runway width, 200 feet, was too narrow for four aircraft to have lateral separation required, for safety considerations, in a simultaneous take-off). For aircraft landing in flights of four (only two observations were made), the landing time per aircraft was about one fourth less than for an aircraft which landed in a flight of two.

Over 50% of the carrier jet timings were for dual take-offs or landings, so in view of the above discussion and to reduce the number of service time distributions in the problem it will be assumed that all jet aircraft depart and land in flights of two.

For propeller type aircraft practically all of the data obtained were on the R6D transport plane. These data will be used to represent the landing and take-off distributions of heavy propeller naval land base aircraft.

When the distribution of the service times of the landings and take-offs were combined, with equal weighting,

for both the jet and the propeller type aircraft, the distributions obtained appear much like negative exponential distributions (see Appendix III, page 42). In considering the service time distribution of the runways over a long period of time a negative exponential distribution will be assumed. The parameter of this distribution will be determined by the ratio of the jet and the propeller aircraft serviced during the interval.

The mean, m, and standard deviation, σ, obtained for jet aircraft operating in pairs and for propeller aircraft are given in Table I below.

Units:	Take-off		Landing		Either Take-off or landing	
Seconds/aircraft	m	σ	m	σ	m	σ
Jet aircraft in pairs	14.2	8.6	44	12.0	29.1	14.8
Propeller aircraft	19.5	10.7	69	17.9	44.3	20.9

Means and Standard Deviations of Data Collected

TABLE I

When the airfield is operating in instrument weather, there is another service time distribution for the airfield. This service time distribution was studied in part by G. E. Bell [6] in 1949. He found that, for transport aircraft at airports around London, the mean approach time

was eight minutes. The approach techniques used were similar to those being used at commercial and military airports in the United States.

The basic definition of a departure and an approach as given in Chapter I will be expanded upon by defining the terms "departure time" and "approach time". Departure time is defined as the time a departing flight commences its take-off, in instrument weather, until the next event can occur; and approach time is defined as the time an approaching flight commences its landing roll, in instrument weather, until the next event can occur. With these definitions it can be seen that the instrument approach control facility might easily have six or eight approaches or departures in progress at the same time.

For military advance airbase operation in times of a national emergency a system requiring 3, 5, or 8 minutes for an approach or departure is too slow. For this reason the present instrument approach control in use at civil and military airbases will not be used; in its place a system having a more acceptable service rate will be used.

No definite system will be described here but it will be assumed that, from the radar approach control and other techniques under consideration by the Civil Aeronautics Authority and the Armed Forces, a system will be devised which is characterized by a distribution of departure times and approach times similar to that for the take-off and

landing times. The service time distribution of the instrument approach control over a long period of time will be assumed to be a negative exponential distribution. The parameters for these distributions are given in Table II below.

Units:	Departure		Approach		Either: Departure Approach	
Seconds/ flight	m	σ	m	σ	m	σ
Jet a/c in pairs or propeller a/c individually	30	15	60	30	45	33.6

Mean and Standard Deviation of Assumed Instrument Facilities

TABLE II

Bell [6] pointed out that the type of distribution of service time assumed has very little effect on the trend of the results. In fact, in his study he assumed the approach time to be constant when the data appeared to have a Pearson type III distribution.

There are many items which will complicate the system. One item of particular importance to the logistics planner is the time lost to change from departures to arrivals or conversely when the airspace available (i.e. clear of terrain hazards) is limited such that the departing and arriving aircraft must utilize the same airspace.

Weather around an airfield very seldom is all good or

all instrument. It is frequently good at very low altitudes but an overcast sky makes instrument departures and approaches through the clouds necessary for many aircraft flights. The actual consequence of this is that the air operations at an airbase is a simultaneous mixture of good and instrument weather operations. To remove this complication only the two extreme conditions (weather all good or all instrument) will be considered.

It is usually considered that the service time of a service facility decreases during periods of high traffic density. Here service times will be considered as dependent only on the type service performed and the type aircraft involved; and that the difference in the average service time obtained when there is other air traffic, and the average service time obtained when a pilot is merely using recommended pilot procedures, taught him during his flight indoctrination, is here assumed to be zero. In other words, it is assumed that the recommended pilot procedures, when correctly followed, will give the least average service time. This is not an unreasonable assumption when it is recalled that generally flight procedures of this type are designed to provide rapid but safe flight operating procedures.

Arrival Interval Distribution

The distribution of arrival intervals at the airbase can best be discussed by considering it in two phases.

These phases are: how the mean arrival interval changes with time, and how the individual arrival intervals vary around any particular mean.

First to be considered is the manner in which the arrivals of aircraft varies around the mean. Bowen and Pearcey [8] collected data on the arrival interval between arrivals at Kingsford-Smith airport, Sidney. Even though the aircraft were scheduled to arrive at definite times the distribution of arrivals fitted the curve of random arrivals (Poisson Distribution of arrivals). This points out that meeting scheduled arrivals for service at an airbase is seldom accomplished. In this paper it will therefore be assumed that arrivals are distributed at random about the mean arrival time.

The mean arrival time $1/\lambda(t)$ of automobiles at a toll booth and many other similar recurring situations have been forcasted quite accurately using analyzed results of statistical data previously collected on the system. Data on the mean arrival time $1/\lambda(t)$ taken at Marine, Naval and Air Force bases in the Far East during the Korean conflict would be useful in forecasting aircraft arrivals when matched with the air groups supported and their required capabilities. Attempts at locating such data have not proven fruitful.

To circumvent these difficulties an alternate approach has been taken; inputs for this approach will be estimated theoretically in the next Chapter. In this approach the mean arrival rate will be considered when the increment of time is 24 hours. This will be done using the assumption that the monthly flight requirements are equally divided among the days of the month. Also the short term mean arrival time necessary for the aircraft groups supported to meet their operational capacity requirement will be considered in Chapter IV.

The discussion just given on arrival time distribution holds true for instrument weather as well as for good weather operations.

Priority System

The priority system at Naval Airbases today is given by the following sequence listed in the order of service.

a. Emergency service.

b. VIP service.

c. Landing or approach service.

d. Take-off or departure service.

At the airbase under consideration the priority system intuitively would be about the same as listed with the addition of (1) operational emergency service and (2) operational service, these items being placed between a) and b) above. Priority operations where the service

mechanism is held open for the arrival of the priority traffic slows down a system. Other than this the net result of priority operations is a rearrangement of the same total waiting time among the different customers. It will be assumed that the priority system used at advanced airbases is merely a rearrangement of the total waiting time.

CHAPTER IV

APPLYING THE MODEL TO THE SYSTEM

In Chapter II it was mentioned that the general solution for the transient state of the system had been solved for only special cases, such as ρ constant. In Chapter III the difficulty of predicting the behavior of the mean arrival time was mentioned. To obtain information on the limiting capabilities of the airbase system, by means which do not require a general solution of equations (1), (2), and (3) and which do not require continuous predictions on the behavior of the mean arrival rate, only the aspects of the system which are most restrictive on air operational requirements will be considered. The possibility that these aspects correctly determine the limits of the system should be reexamined frequently so long as aircraft characteristics and air operating techniques are changing. For the aircraft and air operating techniques of today, three aspects of the system were chosen as being necessary for consideration. They are:

a. The stability of the system. (Can the airbase system handle the traffic assigned with a reasonable small chance of aircraft waiting for long periods of time before being serviced?)

b. The recovery time of the system. (How long does the

last aircraft in a large flight have to wait before it is serviced?)

c. The expected delays associated with a reduced or interrupted service capability. (When a runway is closed by an aircraft accident, what is the maximum expected delay to aircraft associated with the emergency?)

A review of the operational requirements of a combination of aircraft groups to be supported under each of the three conditions listed above will provide the logistics planner with a lower limit on the number of airbase runways and the number of instrument approach controls needed to support the mission requirements of all the planned aircraft groups. When additional instrument approach controls or runways are mentioned as a requirement in this paper, the restraint imposed in the location of the additional site is that it must be separated from the original sites by a sufficient distance to allow the airbases to operate without one interfering with the service facilities of the others. As an example, if three runways are necessary to meet the good weather requirements of a system and one instrument approach control will satisfy the instrument weather requirements, then two sites separated sufficiently to provide non interference among their take-off and landing traffic will suffice. (This assumes that more than two runways are not operationally feasible for simultaneous operations from

one airbase.)

Stability of the System

To determine the steady state condition of the system the average number of arrivals over a long time period (one day will be used in this paper) must be compared with the average service time for the types of traffic being serviced.

By arithmetic manipulations the long time mean arrival rate can be obtained for good weather operations and for instrument weather operations using:

a. The number and type of aircraft groups to be supported.

b. The number of aircraft by type (carrier jet or heavy propeller type) assigned to each group.

c. The expected monthly flight hours per aircraft.

d. The percent of the monthly flight time to be conducted under good weather conditions only.

e. The average length of flights to be conducted under good and instrument weather conditions.

Items a) and b) above are obtainable from the mission statements. Items c), d), and e) are obtainable from operational experience of current aircraft squadrons, type command training requirements, and crew and/or aircraft endurance information.

Next consider the mean service time. A ratio of the number of jet aircraft serviced per day to the number of propeller aircraft serviced can be obtained for both weather conditions. The mean service time of the system is a linear

combination of the mean for the two types of aircraft (jet and propeller) considered, with coefficients determined by the relative frequency of service by each aircraft type. The mean service times for the two basic aircraft types and for the two conditions of operation are given in Tables I and II of Chapter III. Under the assumption made regarding approach or departure times, the mean service rate is independent of basic aircraft type during instrument weather conditions. The mean service time for good weather operations is shown in Figure (1) below for various combinations of jet and propeller aircraft service frequencies.

% jet services

100	80	60	40	20	0
29.1	32.1	35.2	38.2	41.3	44.3

Mean Service Time of the Combination in Seconds/Aircraft

Figure (1)

Using the value of the mean arrival rate, λ, and the mean service time, $1/\mu$, a traffic density value can be obtained:

$$\rho = \frac{\lambda}{n \cdot \mu},$$

where n is a positive integer (number of runways or instrument approach controls) necessary to produce a sufficiently small probability, P/τ, of an aircraft being delayed no

longer than a given small time interval τ. The choice of n used to give an exceptable small chance of a given delay determines the number of runways or instrument approach controls needed for the advanced airbase complex when the expected number of aircraft arriving for service during a given time increment is the same throughout the day. This is a measure of the ability of an airbase system to handle its total daily load.

Values of P/τ can be obtained by using Equation (5) of Chapter II;

$$P/\tau = P/o \cdot F(\rho),$$

andthe graphs of P/o and $F(\rho)$ given in Appendix I.

Recovery Time of the System

Air operations require that an airbase system have the capability of launching a large number of aircraft in a relatively short period. This is needed if it is desired to conduct large mass air strikes against the enemy, and if the fighter aircraft are to defend against simultaneous attacks by the enemy. The short endurance of modern jet aircraft makes the return aspect of a mass attack or large defense effort just as severe a problem. The item of interest here is how long it will take to service a given size flight of aircraft.

To develop a process to determine the answer to the recovery time of the system, let:

$f(t)$ = the density function for a designated service time distribution for a flight of aircraft (e.g., $f_{jT}(t)$ = the density function for jet aircraft take-off times) with $E(t)$ and σ_t as the mean and standard deviation of the distribution.

If there are k similar type aircraft in the flight, and m and σ are the mean and standard deviation of their service time distribution, and if the distributions are independent then the mean and variance of the entire flight of k aircraft are:

$$E(t) = km \tag{8}$$

$$\sigma_t^2 = k\sigma^2 \tag{9}$$

From histograms of observed service times for take-offs and landings, given on page 3 of Appendix III, it is found that only between 18 and 22 percent of the service times are greater than $m + \sigma$. Also the data indicate that the actual distributions of take-offs and landings times for jet aircraft operating in pairs and the R6D aircraft are not particularly skew or bi-model, in fact they are not too different from a normal type distribution. Therefore, it will be assumed that these distributions have reproductive type properties as does the normal distribution. Then it is reasonable to expect that about 80% of the time values of $f(t)$ (service times) would be less than $E(t) + \sigma_t$. Values of

$E(t) + \sigma_t$ for various service operations and number of aircraft in the flight were computed using Equations (8) and (9) above and the results are given in Table III below.

	k	Good Weather Operations: Jet a/c	Good Weather Operations: Propeller a/c	Instrument Weather Operations: Either or both type a/c
takeoff or departure	1	.4	.5	.8
	10	2.8	3.8	5.8
	20	5.4	7.3	11.2
	30	7.9	10.7	16.4
	40	10.4	14.1	21.6
	50	12.9	17.5	26.8
	75	19.0	25.8	39.6
	100	25.1	34.3	52.5
landing or arrival	1	.9	1.4	1.5
	10	8.0	12.5	11.6
	20	15.6	24.3	21.3
	30	23.1	36.1	32.7
	40	30.6	47.9	43.2
	50	38.1	59.6	53.4
	75	56.8	88.9	79.4
	100	75.3	128.0	110.0

Values $E(t) + \sigma_t$ in minutes

for Various Service Operations

and Number of Aircraft in the Flight

TABLE III

When determining the time required for a flight to be serviced it will be assumed that:

a. service times less than $E(t) + \sigma_t$ will provide a satisfactory level of assurance that the service facility can handle the flight of aircraft.

b. other units desiring service during the period can and will be delayed until the flight has been serviced.

c. each additional service facility has the same ability to service aircraft as does one facility alone.

With these assumptions and Table IV the number of facilities necessary to meet the operational requirements of a large flight of aircraft can be determined. For example, if the time required to launch 50 jet aircraft in good weather conditions is of interest, from Table III, it is found that 12.9 minutes are required if one runway is used and about 6.7 minutes if two runways are used.

System Response to a Temporary Reduction in Service Facilities.

The last of the three conditions to be considered is the ability of the system to handle a temporary reduction in the service facility, as when a runway is closed or when the approach channel is held open for an emergency approach. In the first case, where an aircraft is stopped on the runway, the time required to remove it ranges from 15 to 60 minutes*.

* Observations of the writer during a tour of duty as Air Traffic Control Officer at NAS Atsugi, Japan during 1953-55.

In the latter case it will be assumed that the procedures being developed can overcome this difficulty. Let:

τ_e = time that the facility is closed by the emergency.

τ_m = the maximum expected waiting time caused by an emergency of length τ_e.

τ_w = the expected waiting time for an aircraft arriving for service under normal operating conditions

$M_w(t_e)$ = expected number of aircraft waiting for a landing when the runway is cleared at time t_e and when w were waiting when the facility was closed at time t.

w = expected number waiting under normal operating conditions.

For a single runway system, n=1, the maximum expected waiting time for an aircraft arriving during the emergency* is:

$$\tau_m = \tau_w + \tau_e \tag{10a}$$

For a dual runway system, n = 2, assuming that there was no one waiting at time t, the maximum expected waiting

* If the airbase had a number of runways of which only the one nearest to the wind direction was normally used, then this approach holds only when the cause of the emergency stops the use of all the "out-of-the-wind" runways.

time for an aircraft arriving during the emergency is:

$$\tau_m = \frac{M_o\ (t_e)}{\lambda} - \frac{1}{\mu} \qquad (10b)$$

Values of $M_o\ (t_e)$ can be obtained from the graph on page 2 of Appendix II. The value at which τ_m would be considered unacceptable depends, among other things, on the location of nearby runways accessible in all weather conditions and the ability of the reduced system to meet operational requirements. If there is no other runway within twenty or thirty miles a dual runway system will be considered here as a requirement for jet aircraft operations.

CHAPTER V

SOLVING AN EXAMPLE PROBLEM

Chapter IV described a procedure which would provide information on the number of instrument approach controls and runways needed to support adequately the operational demands of the aircraft groups to be supported. This chapter will illustrate by an example the way a logistics planner of an Area Commander's Staff can, from the information given him, determine the minimum number of runways and instrument approach controls needed in the advanced base area.

Suppose that the problem at hand concerns a forthcoming amphibious advance by the United States forces. This is the first in a series of advances and it is desired to develop support facilities for future advances. The land area to be occupied is an island within easy range of enemy bombers. The mission of the naval air advance base unit is to provide support for the following:

3 - patrol squadrons (abbreviated VP).

1 - air early warning squadron detachment (abbreviated VW).

1 - fleet air logistics terminal for transport aircraft turn around (abbreviated VR).

2 - close air support and light attack squadrons (VC).

4 - air defense squadrons (abbreviated VF).

The data given below, in the first five lines of Table IV, were taken as representative of the operating characteristics of the units, after a study was made of past operational procedures for the above aircraft units.

Unit Type	VP	VW	VR	VC	VF	
No. of Units	3	1	1	2	4	
No. a/c per unit	12	4	*	24	24	
Flight hours per a/c month	100	130	*	70	70#	
Average all weather flight duration (operational flights)	12	18		2.5	1.5	
Average good weather only flight duration (training flights)	4	4		1.5	1.5	
Percentage of flight time in good weather only	10	10		10	30	Total
All weather flights per day	9	0.9	8	40.3	104.5	162.7
All weather & good weather flights per day	12	1.3	8	47.8	149.3	218.4

Flight Requirements by Aircraft Units

TABLE IV

* VR requirements are four arriving and four departing flights per day.

Additional requirements for the 4 VF together:

1. The ability to provide all weather take-off service for 24 aircraft in 10 minutes.

2. The ability to provide all weather landing service for 24 aircraft in 20 minutes.

Stability of the System

The data given in the first five lines of Table IV can be combined arithmetically to give the numbers in the last two lines of Table IV.

The average long time traffic density of the system under instrument weather conditions for n = 1 is:

$$\rho = \frac{2 \times 162.7}{24 \times 60} \Big/ \frac{60}{45} = .17$$

Using this value for ρ and $\mu\tau = 3$ (an arbitrary small value) then from the graphs of Appendix I it is found that:

$$P/2.7 = .17 \times .08 = .0136$$

The average long time traffic density of the system under good weather conditions and the probability of an arrival having to wait at least three service time units determined in a similar manner is:

$$\rho = \frac{2 \times 218.4}{24 \times 60} \Big/ \frac{60}{30.6} = .15$$

and

$$P/1.5 = .15 \times .08 = .012$$

The above results indicate that for a one runway -

one instrument approach control airbase system the probability that an arrival aircraft requires three minutes or more is about .01. This would surely be an acceptable situation if this were the only consideration; therefore, one runway and instrument approach control system can easily handle the long time traffic load.

Recovery Time of the System

Next consider the system that is required to satisfy rapid arrival (24 a/c in 20 minutes) and rapid departure requirements (24 a/c in 10 minutes). From Table III, 24 aircraft under all weather conditions require 18.5 minutes to depart and 26.4 minutes to land, so one additional instrument approach control is required to meet the take-off and landing requirements under all weather conditions. It is not necessary to check the good weather ability since for jet aircraft the service time is consistently less than it is for instrument weather conditions.

System Response to a Temporary Reduction in Service Capability

The last consideration is the ability of the runways to handle an emergency. In the previous discussion concerning rapid handling ability it was shown that two approach controls were required. However, an emergency at one of the airbases would reduce the instrument weather rapid handling rate down to 19 aircraft departures in 10 minutes and 18

aircraft arrivals in 20 minutes. This reduction in rate might be difficult to realize (i.e., all ready aircraft are at the airbase with the closed runway). To fully meet the operational requirements during an emergency both instrument approach controls should be supported by dual runways.

Under this condition the determination of τ_m is unnecessary, but it will be determined as an exercise in procedure.

Suppose that 70% of the overall traffic arrive during the 16 hours around the normal day and that the traffic is distributed equally among the two airbases.

Then:

$$\lambda = \frac{218.4}{16 \times 60} = .227 \text{ arrivals/minutes;}$$

and ρ = .116

for a good weather day. Suppose the length of the emergency was 15 minutes ($\mu \cdot t \simeq 29$). From Appendix II it can be seen that the system has reached its steady state condition. Therefore, using Equation (4) of Chapter II:

$$\tau_m = \tau_w = \frac{.227}{.51(.28)} = 1.6 \text{ minutes}$$

In other words the closure of one of the runways at one of the two airbases will only have associated with it an expected waiting time of 1.6 minutes for aircraft arriving for service while the runway is closed.

Example Results

It has just been shown, subject to the assumptions used, that the operational requirements of the aircraft to be supported by the advance bases developed will be fully met only when two dual runway airbases are provided and located so that their instrument approach control systems can operate independently. With this information as a working foundation the logistic planner can now undertake problems of determining site locations and material requirements for the airbases.

BIBLIOGRAPHY

1. Naval Publication — BASE DEVELOPMENT (short title: NWP 11-23) Chief of Naval Operations Publication April 1953

2. Churchman, C. W. Ackoff, R. L. Arnoff, E. L. — INTRODUCTION TO OPERATION RESEARCH John Wiley & Sons, Inc. New York, 1957

3. Molina, E. C. — APPLICATIONS OF THE THEORY OF PROBABILITY TO TELEPHONE TRUNKING PROBLEMS Bell System Technical Journal Vol. VI No.3 July 1927

4. The Port of New York Authority — A COLLECTION OF DELAY PROBABILITY FORMULAS AND CURVES Operations Standards Division The Port of New York Authority March 1956

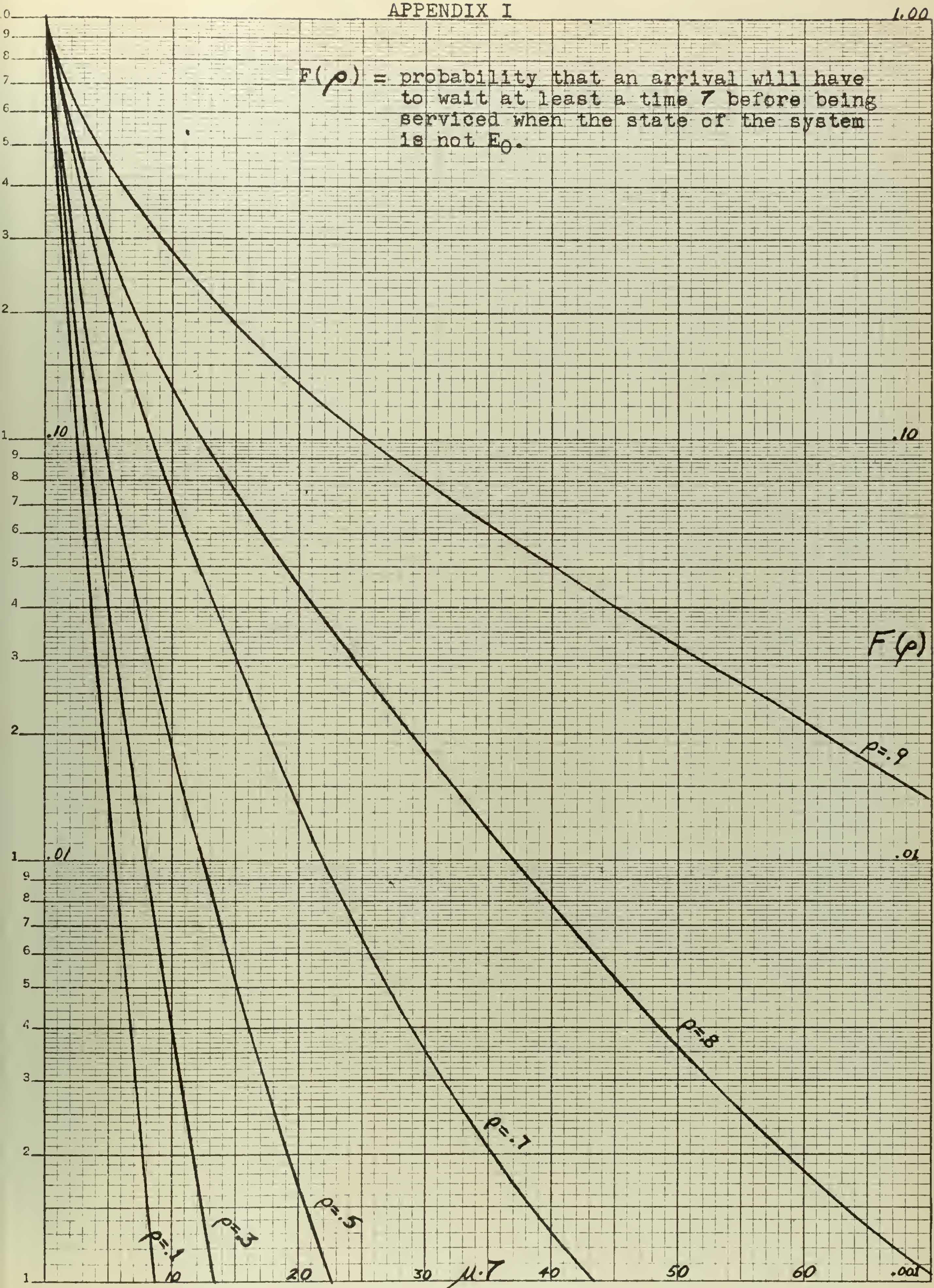
5. Clarke, A. Bruce — THE TIME DEPENDENT WAITING LINE PROBLEM University of Michigan, Ann Arbor March, 1953

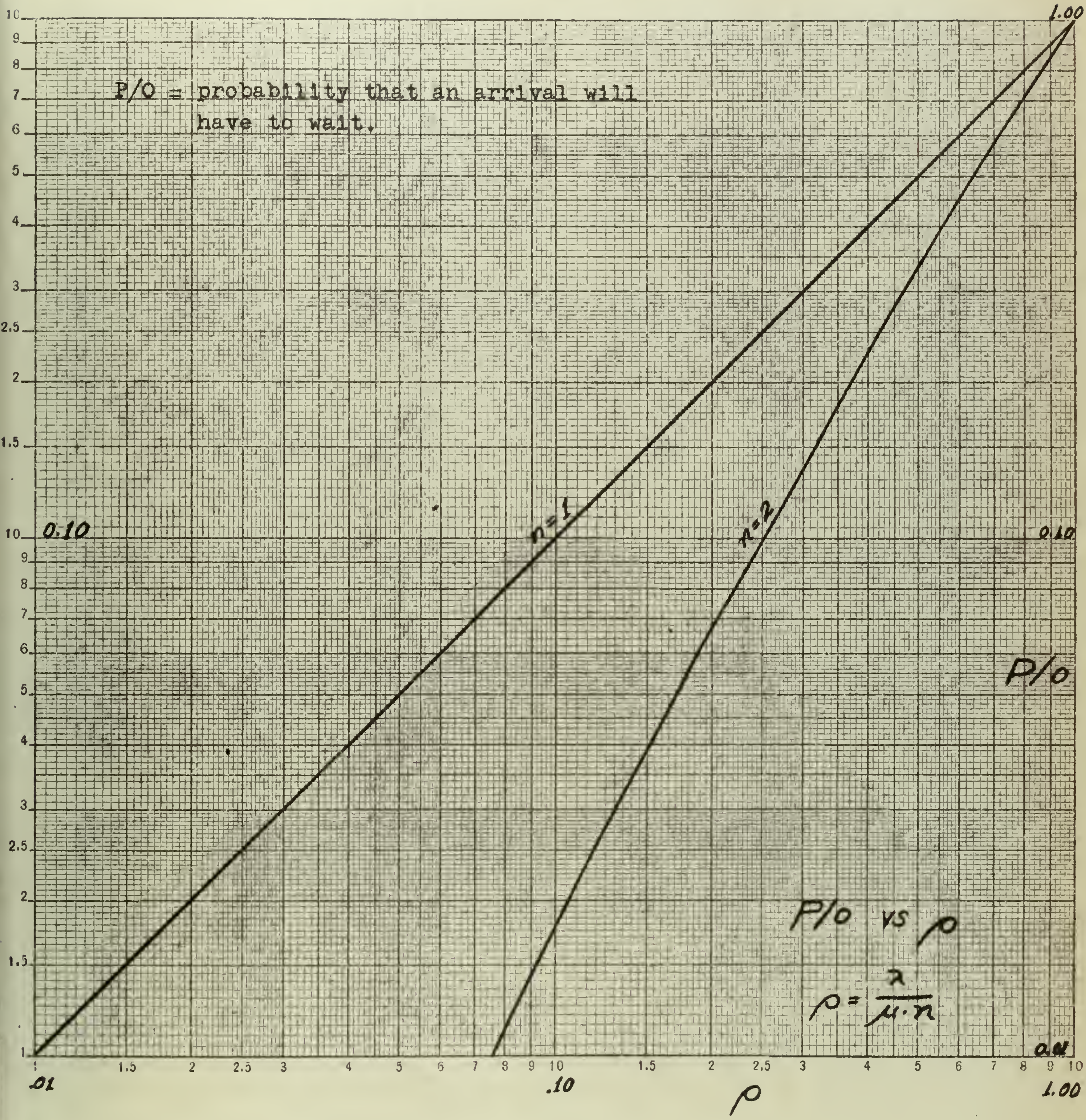
6. Morse, P. M. — STOCHASTIC PROPERTIES OF WAITING LINES Journal of the Operations Research Society of America Vol.3 No.3 August, 1955

7. Luchak, George — THE SOLUTION OF THE SINGLE CHANNEL QUEUING EQUATION CHARACTERIZED BY A TIME-DEPENDENT POISSON DISTRIBUTED ARRIVAL RATE AND A GENERAL CLASS OF HOLDING TIMES Journal of the Operations Research Society of America Vol.4 No.6 December 1956

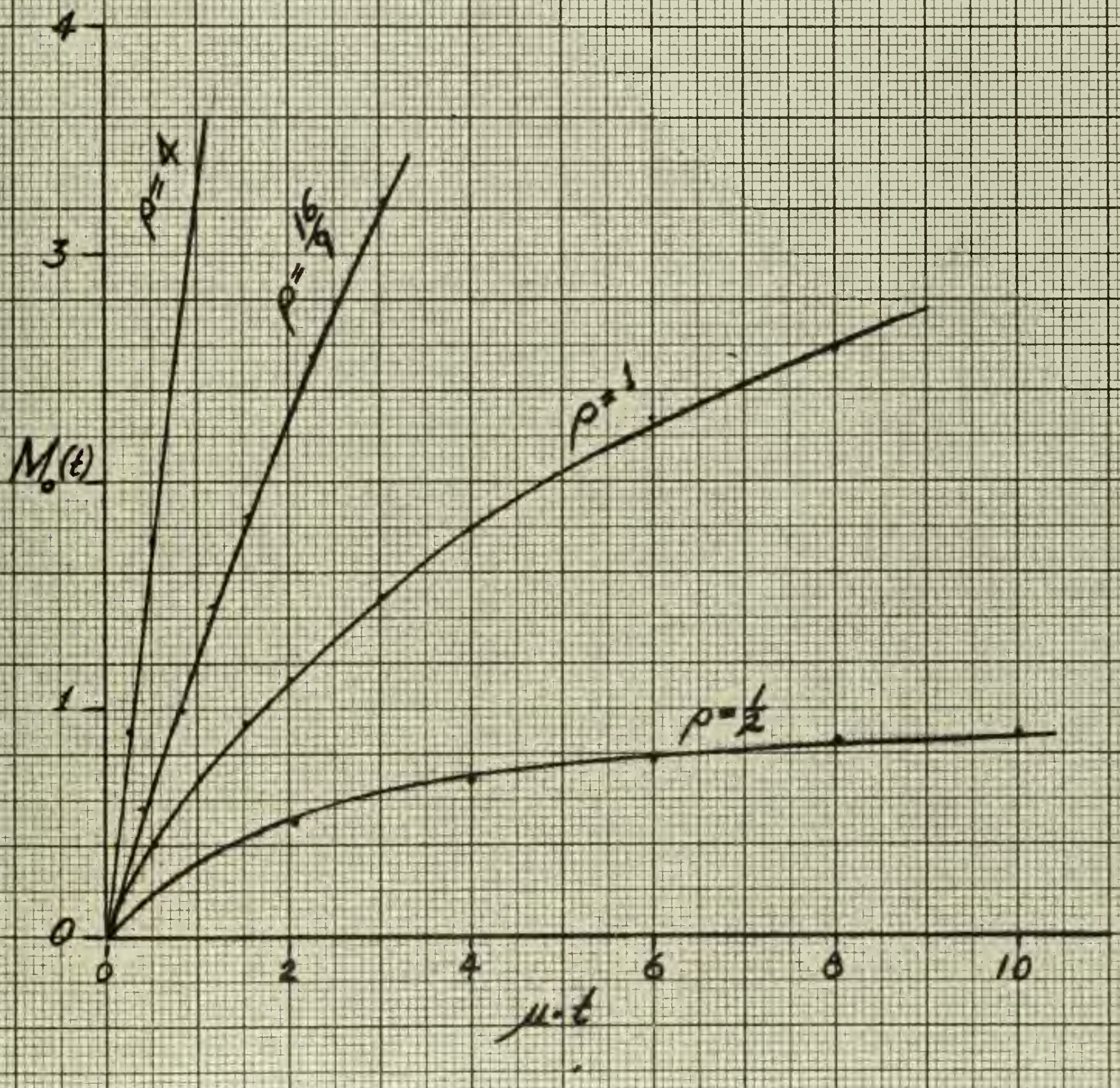
8. Bowen, E. G.
Pearcey, T.

DELAY IN THE FLOW OF THE AIR TRAFFIC
Journal of the Royal Aeronautical Society
Vol.52 No.448 April 1948


1.00
F(ρ) = probability that an arrival will have to wait at least a time 7 before being serviced when the state of the system is not E0.
.10
.10
F(ρ)
ρ=.9
.01
.01
ρ=.8
ρ=.7
ρ=.5
ρ=.3
ρ=.1
10
20
30
μ·7
40
50
60
.001


P/O = probability that an arrival will have to wait.
n=1
n=2
P/o
P/o vs ρ
ρ = λ / (μ·n)
.01
.10
1.00
0.10
0.01
ρ

$M_o(t)$ = the expected state of the system at time t when the system started in state zero.



APPENDIX III

Aircraft Take-off and Landing Time Data

Two hundred eighty-eight timings were made of aircraft taking off and landing at NAS Moffett Field at varying intervals on 11, 12, 14, and 15 of March, 1957. The runway wind conditions varied from a 5 knot tail wind to a 17 knot head wind. The runway surface was wet only during one 70-minute period on 12 March.

Moffett Field has a dual 8000 by 200 foot runway at approximately sea level. There are numerous perpendicular turn-off taxi-ways spaced along the side of both runways. There are no cross runways.

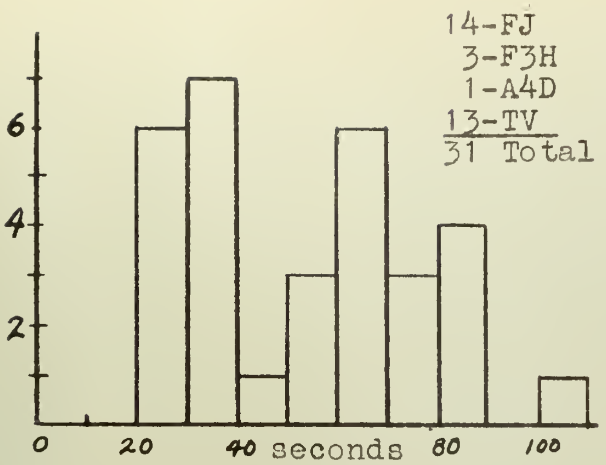
A take-off time was taken as the time at which the aircraft was cleared to take-off until the aircraft was airborne. Frequently aircraft were cleared to take-off when in the aircraft warm up position, off the runway. In these cases two items were timed, the time required for the aircraft to get lined up ready to take-off and the time required to take-off. The measured value of time to line up indicated that there was sufficient time after a landing aircraft had passed the warm up spot for the take-off aircraft to get lined up and ready to take-off before the landing aircraft cleared the up-wind end of the runway. It also showed that for a take-off to follow a take-off as soon as possible, the second take-off aircraft must be cleared onto

the runway before the first aircraft is cleared to take-off.

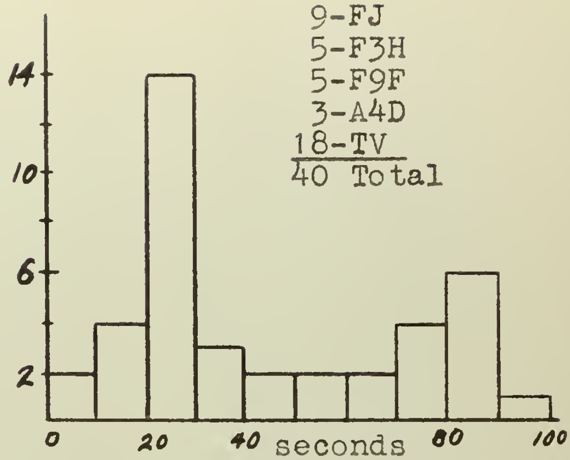
A landing time was taken as the time on the final approach at which the pilot could no longer prevent the aircraft from touching the runway until the aircraft cleared the runway.

In timing a flight of aircraft, the time for one aircraft was the time required for the total flight divided by the number of aircraft in the flight.

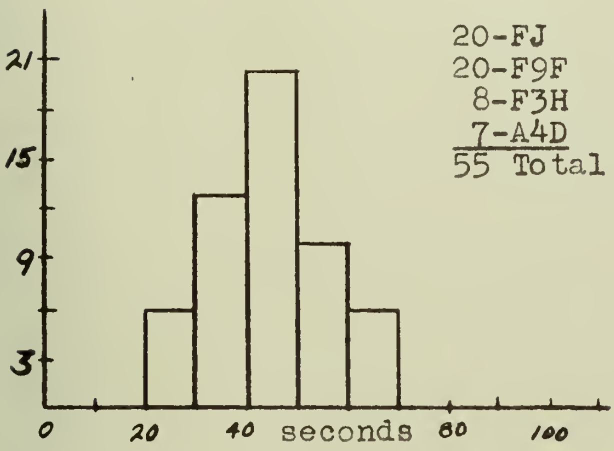
A summary of the data is given on the next page in the form of frequency histograms. The last page of this appendix contains two combinations of the relative frequency histograms of the data given on the next page and is used to show the nature of the overall service time distribution of the runway.



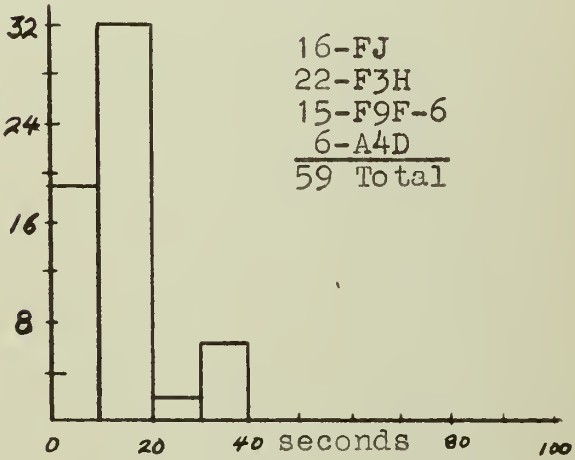
Jets landing independently



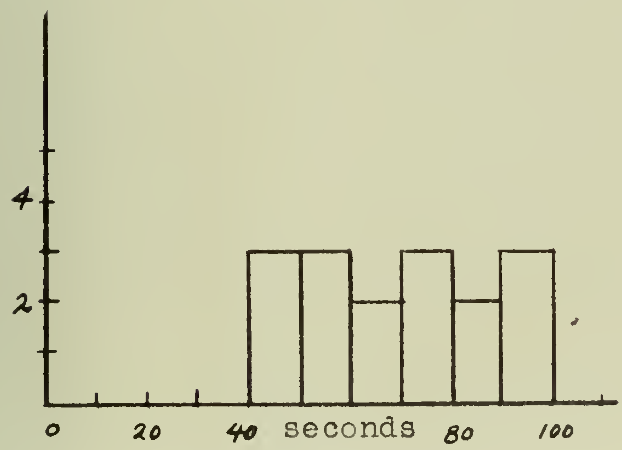
Jets taking-off independently



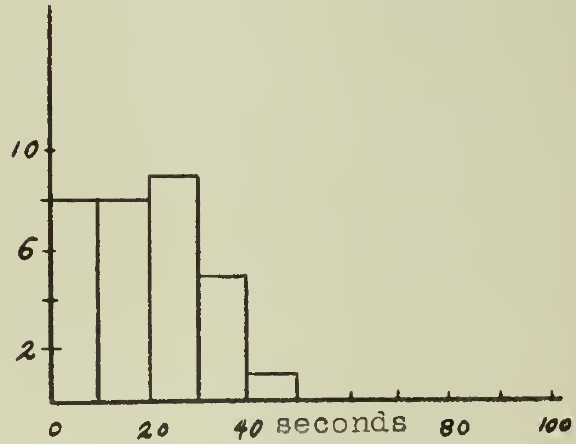
Jets landing in pairs



Jets taking-off in pairs

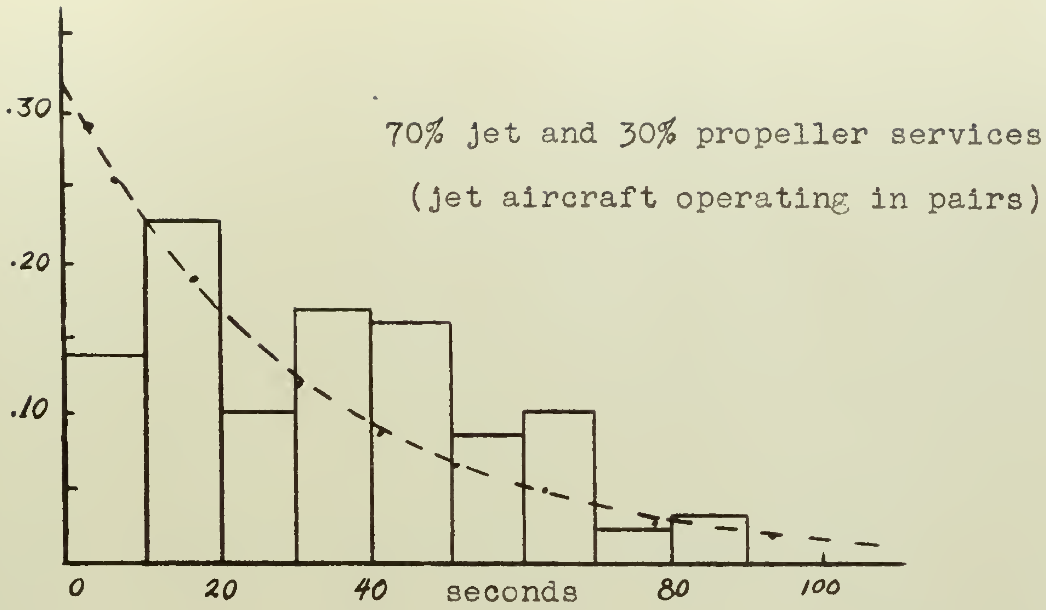


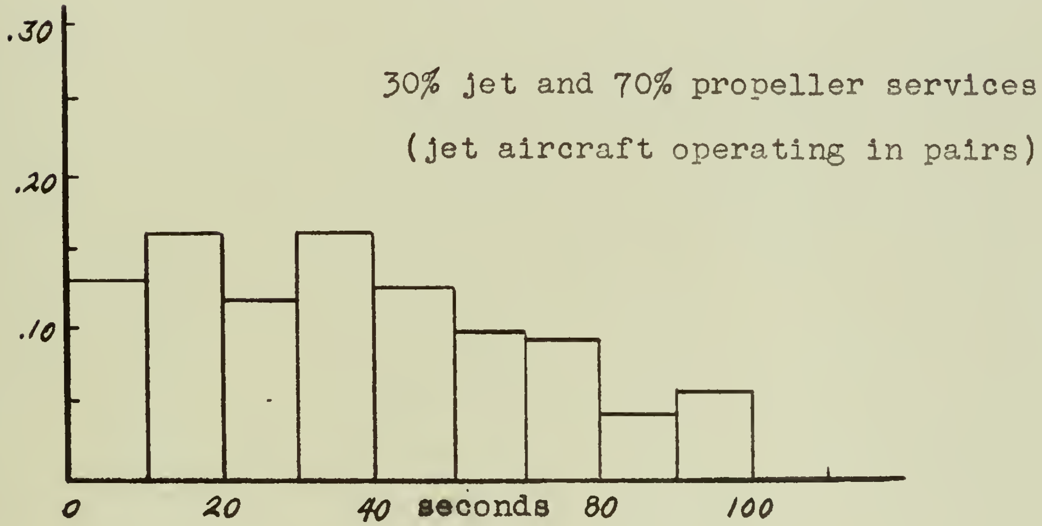
R6D's landing



R6D's taking-off

Histograms of Frequency of events vs time required in seconds/aircraft





Histograms of relative frequency vs time in seconds per aircraft. Landing and take-offs are combined with equal weights. (Dashed curve is a negative exponential curve.)

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